Analysis of the Practical Coverage of Uniform Motions to Approximate Real Camera Shakes

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ABSTRACT

Motion blur is usually modeled as the convolution of a latent image with a motion blur kernel, and most of current deblurring methods limit types of motion blurs to be uniform with the convolution model. However, real motion blurs are often non-uniform, and in consequence the methods may not well remove real motion blurs caused by camera shakes. To utilize the existing methods in practice, it is necessary to understand how much the uniform motions (i.e., translations) can approximate real camera shakes. In this paper, we mathematically analyze motion extents of a camera's non-uniform motion at the optical axis and image boundary, then derive the practical coverage defined as the extents at the two regions remain near uniform. The coverage can effectively guide how much one can utilize the existing uniform motion deblurring methods, and informs the need to model real camera shakes accurately rather than assuming uniform motions.

Keywords: Motion blur, analysis, uniform motion, non-uniform motion

1. INTRODUCTION

Motion blur is a common artifact which severely degrades image quality by losing sharp deatils. Motion blur easily occurs when taking pictures under low-light conditions as cameras need to increase the exposure time of the sensors to acquire enough lights.

For the image deblurring problem, tremendouse progress has been achieved in recent years. However, the deblurring problem is still challenging since the number of unknowns exceeds the number of observed data. Most of existing deblurring methods have assumed uniform (i.e., space-invariant) motion blurs. Fergus et al.[1] proposed a variational Bayesian method on top of natural image statistics for image deblurring. Shan et al.[2] proposed an effective deblurring method using a series of optimization techniques with a simplified representation of natural image statistics. Cho and Lee[3] proposed a practical fast deblurring method which effectively predicts gradients of the sharp latent image using simple image filters. Xu and Jia[4] analyzed negative effects of small scale image structures in kernel estimation and proposed a robust deblurring method which excludes such small scale objects when estimating the blur kernel. However, despite the success of the previous image deblurring methods[1–4] in many cases, their assumption on the blur model can handle only space-invariant motion blurs, thus that limits the methods from estimating real camera shakes.

A motion blur is generally modeled as

$$b = k * l + n, \tag{1}$$

where b is the observed blurred image, k is the motion blur kernel, $*$ is the convolution operator, l is the latent image, and n is image noise. Although the convolution operator $*$ is effective in modeling space-invariant motion blurs caused by the camera's translation, it is difficult to represent space-variant motion blurs such as camera shakes. Thus, the previous deblurring methods based on Eq. (1) are barely suitable for estimating and removing real camera shakes since the fundamental assumption on the blur model does not fit to the real camera shakes. In other words, to utilize the existing deblurring methods in practice, we first need to understand how much

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they can estimate real camera shakes, then apply the methods to real blurred images within their coverage to get effective results.

In this paper, we first analyze motion extents of a non-uniform motion at the image center and boundary, and propose a practical coverage using the difference of the motion extents in which the assumption of the previous deblurring methods on blur model holds true, thus they can be utilized effectively. To do this, we assume that the photographed scene is planar and static, use the pinhole camera model, and adopt the projective motion blur model proposed by Tai et al.[5] to approximate a real camera shake. As we use the non-uniform motion blur model of Tai et al., we can represent a camera shake as a weighted sum of discrete temporal non-uniform motions. Following this direction, the analysis in this paper is conducted for each temporal non-uniform motion. The analysis result of this paper guides how to apply existing uniform motion deblurring methods, and suggests which should be further considered for the existing methods to deal with real camera shakes.

The remaining of this paper is organized as follows. In Sec. 2, we introduce the projective motion blur model of Tai et al. to represent camera shakes more accurately than Eq. (1), and analyze motion extents of a camera's non-uniform motion at the rotation axis and image boundary. Then, based on the analysis, we propose a practical coverage where the difference is minimized. In Sec. 3, we show the effect of the coverage through experimental results. In Sec. 4, we discuss limitations of our analysis and how to apply our analysis for real camera shakes, and conclude the paper.

2. ANALYSIS

2.1 Non-uniform motion blur model

Assuming there is no moving object and the captured scene contains no depth variations, the uniform motion blur model in Eq. (1) can be expressed as:

$$b(\mathbf{x}) = \frac{1}{S}\sum_i l(\mathbf{T}_i\mathbf{x}), \tag{2}$$

where b and l are blurred and latent images, respectively, i is a time instance during the camera's exposure, S is the total duration of the camera's exposure time, $\mathbf{x}$ is a pixel position, $\mathbf{T}_i$ is a transformation matrix that produces a $2D$ translation of the latent image l at time i.

If a camera shake contains only translational motions, Eqs. (1) and (2) are simply different representations of the same model. However, if there exist non-translational motions in a camera shake, it is difficult to extend Eq. (1) to incorporate such non-uniform motions. On the other hand, if we replace $\mathbf{T}_i$ by a general homography $\mathbf{H}_i$ in Eq. (2), then we can obtain a non-uniform motion blur model for a camera shake proposed by Tai et al.[5]

$$b(\mathbf{x}) = \frac{1}{S}\sum_i l(\mathbf{H}_i\mathbf{x}), \tag{3}$$

where $\mathbf{H}_i$ is a homography at a time instance i during the camera's exposure. This projective motion blur model can represent not only the uniform motion blur models in Eqs. (1) and (2), but also more general camera shakes such as camera's rotational motions. See Tai et al.[5] for more details. Furthermore, each homography $\mathbf{H}_i$ in Eq. (3) can be decomposed as:

$$\mathbf{H}_i = \mathbf{K}(\mathbf{R}_i + \mathbf{T}_i)\mathbf{K}^{-1}, \tag{4}$$

where $\mathbf{K}$ is a camera's internal calibration matrix, and $\mathbf{R}_t$ and $\mathbf{T}_t$ are 3×3 rotation and translation matrices, respectively. Note that the projective motion blur model in Eq. (3) can fully represent non-uniform motion blurs caused by a camera's shakes through combinations of rotations and translations in Eq. (4).

In this paper, based on Eqs. (3) and (4), we analyze motion extents of a camera's rotational motions at the optical axis and image boundary, then propose a practical coverage where the uniform motion blur model in Eq. (1) holds true. Note that, our analysis is based on the extent of a single non-uniform motion although a camera shake represented by Eq. (3) consists of i non-uniform motions. In other words, we assume the non-uniform motion analyzed in this paper is of the maximum length among all motions which compose the camera shake,

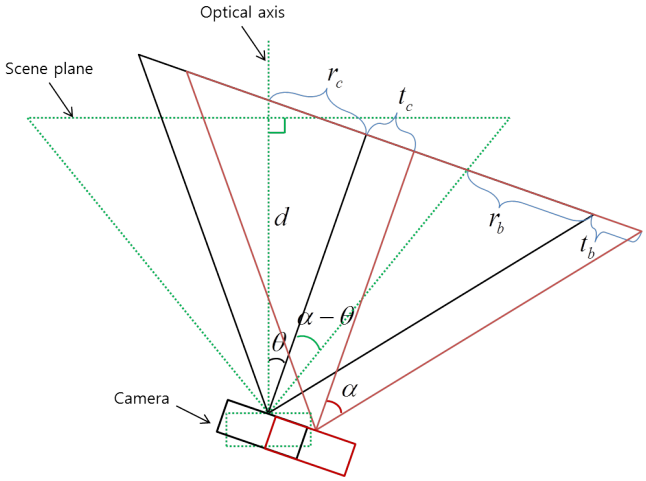


Figure 1. Non-uniform motion represented by $\mathbf{H}_t$ at a single time instance. From the initial orientation (green dotted line), the camera is first rotated (black dash line) then translated (red dash line). The red dash line represents the camera at the final orientation.

and all the other motions are bounded by the analyzed non-uniform motion of the maximum length. In the following sections, we separate $\mathbf{R}_i$ in Eq. (4) of a non-uniform motion into an out-of-plane rotation (so called yaw or pitch) and an in-plane rotation (so called roll) according to the characteristic of rotations, and conduct our analysis on each type of rotations.

2.2 Case 1: out-of-plane rotation

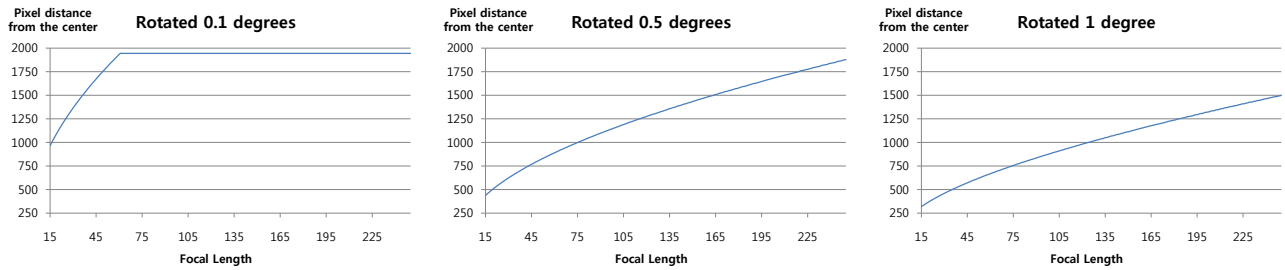
It is worth mentioning first that we assume the following to compute the practical coverage of uniform motions on image pixels (see Fig. 1).

- A practical coverage of a uniform motion is defined as a region of image pixels.
- In this paper, we consider the effect of a uniform motion as the maximum difference of motion extents at the optical axis and image boundary is less than one pixel.
- The scene plane captured by a camera is discretized to the sensor resolution regardless of the real distance between the camera and the scene. In this sense, d is not a real depth, but the number of pixels which is determined according to the sensor resolution. Thus, $d\tan(\alpha) \times 2$ is represented as the same number of pixels to the sensor resolution.
- Similarly to the above, d is represented as the same numer of pixels to the focal length f, and the focal length f can be converted into pixel units using the sensor's pixel density $pixel/mm$,

In Eq. (4), each non-uniform motion (i.e., $\mathbf{H}_i$) at a single time instance i is represented by a rotation followed by a translation. Assuming the out-of-plane rotation (i.e., yaw or pitch) occurs with θ degrees about the optical axis (see Fig. 1), the motion extents of the rotation at the optical axis and image boundary, r_c and r_b, respectively, are defined as:

$$r_c = d\tan(\theta), \tag{5}$$

$$\begin{aligned} r_b &= d\tan(\alpha) - d\tan(\alpha-\theta) \\ &= d\left(\tan(\alpha) + \tan(-\alpha+\theta)\right) \\ &= d\tan(\theta)\left(1 - \tan(\alpha)\cdot\tan(-\alpha+\theta)\right) \\ &= r_c\left(1 + \tan(\alpha)\cdot\tan(\alpha-\theta)\right), \end{aligned} \tag{6}$$



(a) (b) (c)

Figure 2. Focal length vs. maximum pixel distance from the center where the rotation (i.e., yaw) affects uniformly. (a), (b), and (c) correspond to the given rotation angle θ =0.1, 0.5, and 1 degree, respectively.

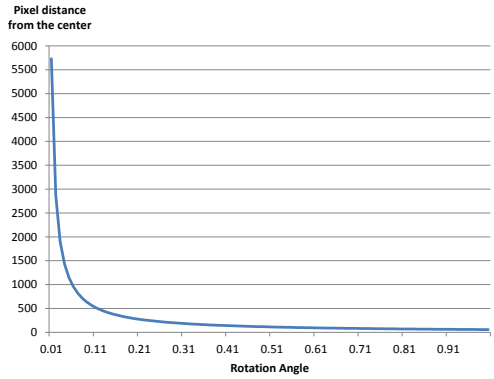


Figure 3. Rotation angle vs. maximum pixel distance from the center where the rotation (i.e., roll) affects uniformly.

where α is the half of a camera's angle of view. Since the extent of a translational motion at the optical axis, t_c, and that at image boundary, t_b, are equal, the difference D_o between the extents of a non-uniform motion at the optical axis and image boundary is defined as

$$\begin{aligned} D_o &= (r_b + t_b) - (r_c + t_c) \\ &= r_c \left(\tan(\alpha) \cdot \tan(\alpha - \theta)\right). \end{aligned} \tag{7}$$

To utilize the uniform motion blur model in Eq. (1) to estimate out-of-plane rotations, we should find α of which the difference D_o in Eq. (7) is less than one pixel. Fixing θ, we can compute the range of α where the difference D_o is less than one pixel. Then, the region of the scene plane in pixel units, which is covered by the maximum value of the angle of view α, is the practical coverage of a uniform motion.

Fig. 2 shows the relationship between the focal length and the number of pixels within the practical coverage of a uniform motion assuming a full-frame DSLR whose image sensor is of the same size as a $35mm(36 \times 24mm)$ film frame, and whose resolution is 3888×2592. According to Fig. 2a, if $\theta = 0.1$, then about 46.04% and 57.57% of the image have uniform motions when the focal lengths are $28mm$ and $35mm$, respectively.

Eq. (7) mathematically explains the above result as the difference between the motion extents is proportional to the value of the tangent function of α, and α increases as the focal length decreases. In other words, if the focal length is shorter, then uniform motions are quite distinct from the out-of-plane rotations (i.e., yaw or pitch). So, in this case, the previous uniform motion deblurring methods are not suitable. On the other hand, they will be more naturally applicable if the focal length becomes longer. To utilize the previous deblurring methods[1–4] for images of short focal lengths, we should somehow crop the images considering the practical coverage. In Sec. 3, we show the effect of the practical coverage through experimental results.

2.3 Case 2: in-plane rotation

The motion extent of the in-plane rotation on the optical axis is determined by the distance k from the rotation axis. At the rotation axis, there is no motion extent because the distance k is zero. Then, since a non-uniform

Figure 4. Deblurring results on out-of-plane rotation and translation: (a) input image, (b) synthetic input blurred image containing both out-of-plane rotation and translation, (c) uniform motion deblurring result using the whole image,[3] (d) cropped image of (b) considering the practical coverage, (e) uniform motion deblurring result using only valid area. We used same parameters for (c)&(e).

motion at a time instance i, $\mathbf{H}_i$, is represented by a rotation followed by a translation, the motion extents of the translation at the rotation axis (i.e., optical axis) and image boundary are equal. Thus, the difference D_i between the extents at the rotation axis and image boundary is defined as

$$D_i = 2\pi k \times \frac{\theta}{360}, \tag{8}$$

where θ is the angle of the in-plane rotation on the axis.

As Eq. (8) contains only two variables θ and k, the focal length does not affect the in-the plane rotation. Besides, the motion extent of the in-plane rotation is proportional to k. Thus, to utilize the uniform motion blur model in Eq. (1) to estimate in-plane rotations, we should find k of which the difference D_i in Eq. (8) is less than one pixel.

Fig. 3 shows the relationship between the angle of the in-plane rotation and the number of pixels k within the practical coverage of a uniform motion. If $\theta = 0.1, 0.5$, and 1.0, then the corresponding k in pixel units are $572, 131$, and 57 *pixels*, respectively. Note that the in-plane rotation becomes quite different from the uniform motion as θ becomes larger. Thus, to utilize the previous uniform motion deblurring methods,[1–4] one should extend the uniform motion blur model to incorporate the in-plane rotation explicitly.

3. EXPERIMENTS

In this section, we show the effect of the practical coverage analyzed in Sec. 2 through experiments. In Fig. 4, (a) is the original image, (b) is the blurred image containing a camera shake which consists of out-of-plane rotations (yaw with $-0.5 \leq \theta \leq 0.5$) and translations. The camera shake is simulated artificially as its motions smoothly change. (c) is the result of Cho and Lee[3] using (b). Since (b) contains rotational motions as well as translational motions, Cho and Lee's method which is based on Eq. (1) could not restore a satisfactory result. To apply the method correctly, we cropped (d) according to the practical coverage from (b). (e) is the result of Cho and Lee using (d). The cropped image (d) is selected within the practical coverage, thus Cho and Lee's method could restore a sharp image.

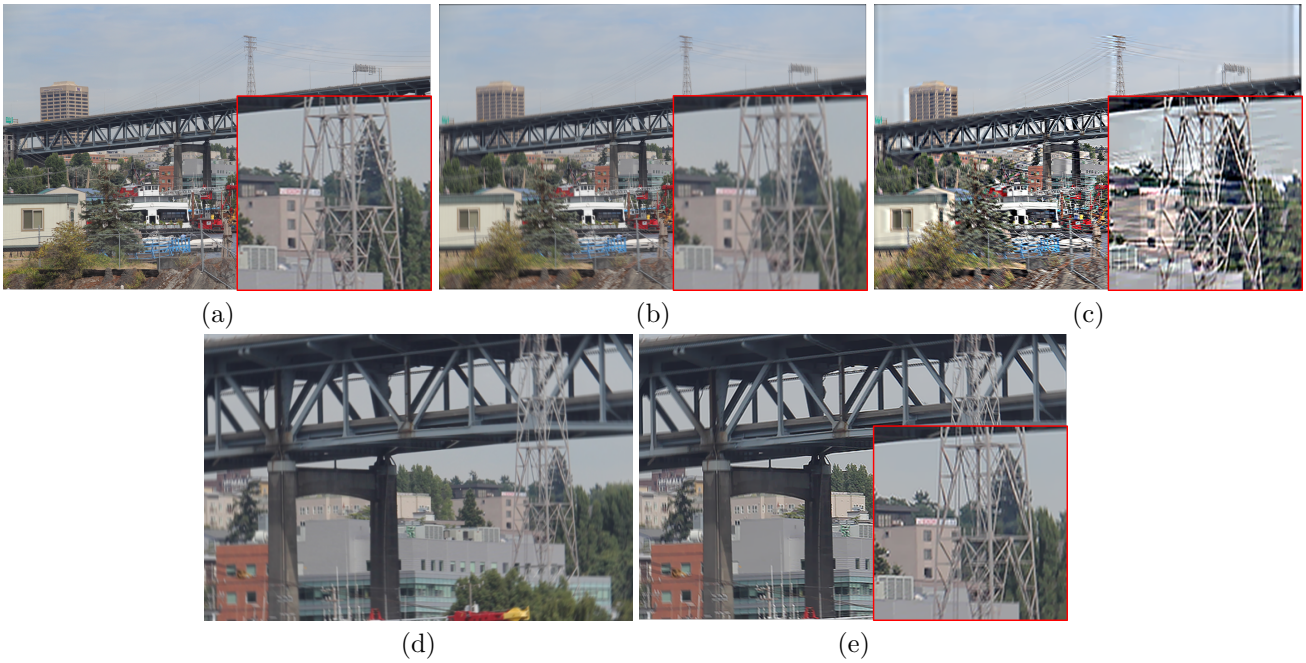


Figure 5. Deblurring results on in-plane rotation and translation: (a) input image, (b) synthetic input blurred image containing both in-plane rotation and translation, (c) uniform motion deblurring result using the whole image,[3] (d) cropped image of (b) considering the practical coverage, (e) uniform motion deblurring result using only valid area. We used same parameters for (c)&(e).

In Fig. 5, the blurred image contains a camera shake which consists of in-plane rotations (roll with $-0.5 \leq \theta \leq 0.5$) and translations. In Fig. 5c, Cho and Lee's method fails because the combination of in-plane rotations and translations is hardly fit to the uniform motion blur model. However, in Fig. 5e, we could get a sharper image with a cropped image (d) because (d) does not contain ambiguous motions for Cho and Lee's method.

4. DISCUSSION AND FUTURE WORK

In this section, we discuss the limitations of the analysis and how to apply for real camera shakes. First of all, we assumed the separable rotations (i.e., out-of-plane and in-plane rotations) to simplify the complicated real camera motions. However, real camera shakes may contain combinations of out-of-plane and in-plane rotations, and computing the exact extent of such complicated non-uniform motions is a difficult task. Thus, the practical coverage in this paper provides a rough boundary for uniform motions to approximate real camera shakes. Secondly, we assumed the pinhole camera model. However, real cameras use lenses and may have different characteristics to the pinhole camera. Although we provide a practical coverage on top of assumptions for simplifying our analysis, one can compute a more accurate coverage considering the real camera model.

To utilize the analysis in this paper in practice, we first need to classify the type of rotations from the observed blurred image and identify the rotation axis. This would be possible through inertial measurement sensors[6] or capturing multiple images to align and estimate homographies.

Lastly, since the out-of-plane rotations (i.e., yaw and pitch) are similar to translational motions, one can use a larger coverage than the practical coverage in Sec. 2 if the focal length is large enough. Besides, one can mitigate the "less than one pixel" condition (e.g., less than three pixels) because it might be a tight boundary for uniformity. In case of the in-plane rotations, the blur model should explicitly represent them because they are hardly approximated by the uniform motions.

In this paper, we analyzed the difference of motion extents at the rotation axis and image boundary, then proposed the practical coverage where the difference is near uniform. This analysis result can provide a guide for previous deblurring methods to be applied properly in practice. We also demonstrated the effect of the practical

coverage through experimental results. Our future work includes more accurate and sophisticated analysis using real camera models, and develop a method for modeling non-uniform motions efficiently on top of the analysis results.

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